Modified April 24, 2012

Radnor High School
Course Syllabus
Advanced Placement Calculus BC
0460

Credits: 1
Weighted: Yes
Length: Year
Format: Meets daily

Grades: 11, 12
Prerequisite: Recommended by Department

I. Course Description

Calculus BC is primarily concerned with developing the student's understanding of the concepts of calculus and providing experience with its methods and applications. The course emphasizes a multi-representational approach to calculus, with concepts, results, and problems being expressed graphically, numerically, analytically, and verbally. The connections among those representations are also important. The focus of the course is neither manipulation nor memorization of an extensive taxonomy of functions, curves, theorems, or problem types. Although they are important outcomes, they are not the core of the course. Through the use of unifying themes of derivatives, integrals, limits, approximation, and applications and modeling, the course becomes a cohesive whole rather than a collection of unrelated topics. These themes will be developed using all the functions studied in previous mathematics courses.

II. Materials & Equipment

Calculus, Seventh Edition, Anton, Bivens, Davis; Wiley (2002)
TI-84 Plus or TI-89 Graphing Calculator Required

III. Course Goals & Objectives

. Students should be able to work with functions represented in a variety of ways: graphical, numerical, analytical, or verbal. They should understand the connections among these representations.
. Students should understand the meaning of the derivative in terms of a rate of change and local linear approximation and should be able to use derivatives to solve a variety of problems.
. Students should understand the meaning of the definite integral both as a limit of Riemann sums and as the net accumulation of change and should be able to use integrals to solve a variety of problems.
. Students should understand the relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus.
. Students should be able to communicate mathematics both orally and in well-written sentences and should be able to explain solutions to problems.
. Students should be able to model a written description of a physical situation with a function, a differential equation, or an integral.
. Students should be able to use technology to help solve problems, experiment, interpret results, and verify conclusions.
. Students should be able to determine the reasonableness of solutions, including sign, size, relative accuracy, and units of measurement.
. Students should develop an appreciation of calculus as a coherent body of knowledge and as a human accomplishment.

IV. Course Topics (Summary Outline)

I. Functions, Graphs, and Limits

Analysis of graphs: With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

Limits of functions (including one-sided limits)

- An intuitive understanding of the limiting process
- Calculating limits using algebra
- Estimating limits from graphs or tables of data

Asymptotic and unbounded behavior

- Understanding asymptotes in terms of graphical behavior
- Describing asymptotic behavior in terms of limits involving infinity
- Comparing relative magnitudes of functions and their rates of change (for example, contrasting exponential growth, polynomial growth, and logarithmic growth)

Continuity as a property of functions

- An intuitive understanding of continuity. (The function values can be made as close as desired by taking sufficiently close values of the domain.)
- Understanding continuity in terms of limits
- Geometric understanding of graphs of continuous functions (Intermediate Value Theorem, Mean Value Theorem, Extreme Value Theorem)
- Parametric, polar, and vector functions. The analysis of planar curves includes those given in parametric form, polar form, and vector form.

II. Derivatives

Concept of the derivative

- Derivative presented graphically, numerically, and analytically
- Derivative interpreted as an instantaneous rate of change
- Derivative defined as the limit of the difference quotient
- Relationship between differentiability and continuity

Derivative at a point

- Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
- Tangent line to a curve at a point and local linear approximation, including Newton's Method for approximating zeros
- Instantaneous rate of change as the limit of average rate of change
- Approximate rate of change from graphs and tables of values

Derivative as a function

- Corresponding characteristics of graphs of f and f'
- Relationship between the increasing and decreasing behavior of f and the sign of f'
- *Relationship between domain of a function and the domain of the derivative*
- The Mean Value Theorem and its geometric consequences
- Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

Second derivatives

- Corresponding characteristics of the graphs of f, f', and f''
- Relationship between the concavity of f and the sign of f''

- Points of inflection as places where concavity changes

Applications of derivatives

- Analysis of curves, including the notions of monotonicity and concavity
- Analysis of planar curves given in parametric form, polar form, and vector form, including velocity and acceleration
- Optimization, both absolute (global) and relative (local) extrema
- Modeling rates of change, including related rates problems
- Use of implicit differentiation to find the derivative of an inverse function
- Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration
- Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations
- Numerical solution of differential equations using Euler's method
- L'Hospital's Rule, including its use in determining limits and convergence of improper integrals and series

Computation of derivatives

- Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions
- Basic rules for the derivative of sums, products, and quotients of functions
- Chain rule and implicit differentiation
- Derivatives of parametric, polar, and vector functions

III. Integrals

- Interpretations and properties of definite integrals
- Definite integral as a limit of Riemann sums
- Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval:

$$\int_a^b f'(x)dx = f(b) - f(a)$$

- Basic properties of definite integrals (examples include additivity and linearity)
- **Applications of integrals.** Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen, the emphasis is on using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. To provide a common foundation, specific applications will include using the integral of a rate of change to give accumulated change, finding the area of a region (including a region bounded by polar curves), the volume of a solid with known cross sections, the volume of solids of revolution using discs, washers, and shells, the average value of a function, the distance traveled by a particle along a line, and the length of a curve (including a curve given in parametric or polar form).

Fundamental Theorem of Calculus

- Use of the Fundamental Theorem to evaluate definite integrals
- Use of the Fundamental Theorem to represent a particular anti-derivative, and the analytical and graphical analysis of functions so defined
- Extended study of functions defined as integrals including domain, derivatives, chain rule, and graphical analysis

Techniques of anti-differentiation

- Anti-derivatives following directly from derivatives of basic functions
- Anti-derivatives by substitution of variables (including change of limits for definite integrals), parts, simple partial fractions (linear, repeating linear, and quadratic factors), and trigonometric substitutions
- Improper integrals (as limits of definite integrals)

Applications of anti-differentiation

- Finding specific anti-derivatives using initial conditions, including applications to motion along a line Solving separable differential equations and using them in modeling (in particular, studying the equation $y' = ky$ and exponential growth)
- Solving logistic differential equations and using them in modeling
- Solving first order linear differential equations (integrating factors)

Numerical approximations to definite integrals. Use of Riemann sums (using left, right, and midpoint evaluation points) and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values

IV. Polynomial Approximations and Series

Concept of series. A series is defined as a sequence of partial sums, and convergence is defined in terms of the limit of the sequence of partial sums. Technology can be used to explore convergence or divergence.

Series of constants

- Motivating examples, including decimal expansion
- Geometric series with applications
- The harmonic series
- Alternating series with error bound
- Terms of series as areas of rectangles and their relationship to improper integrals, including the integral test and its use in testing the convergence of p-series
- The ratio test for convergence and divergence
- Comparing series to test for convergence or divergence

Taylor series

- Taylor polynomial approximation with graphical demonstration of convergence (for example, viewing graphs of various Taylor polynomials of the sine function approximating the sine curve)
- Maclaurin series and the general Taylor series centered at $x = a$
- Maclaurin series for the functions e^x, $\sin x$, $\cos x$, and $\frac{1}{1-x}$.
- Formal manipulation of Taylor series and shortcuts to computing Taylor series, including substitution, differentiation, anti-differentiation, and the formation of new series from known series
- Functions defined by power series
- Radius and interval of convergence of power series
- Lagrange error bound for Taylor polynomials

V. Assignments & Grading

Assignment sheets will be distributed periodically throughout the school year. Homework will be assigned on a daily basis. Grades will be based on quizzes and tests. In addition, teachers may use homework, group activities, and/or projects for grading purposes. All students will take departmental midyear and final exams. The Radnor High School grading system and scale will be used to determine letter grades.

Special Notes to Students:

Throughout the year, you will be receiving copies of questions used on past AP exams to complete for homework and/or as test questions. For the AP questions used as homework, you will be required to provide written explanations for all of your work for all parts of each question using correct vocabulary and applicable theorems when appropriate in your explanations. In addition, you will be called upon periodically to orally explain your rationale and your thinking in solving any of these problems as well as other problems used for homework. This will include being called upon to present your work in front of the class.
You are required to have a graphing calculator for this course. Please refer to your hand-out to see the list of AP approved calculators. A TI-84 will be used for instruction. Because much of the work we do involves investigating and analyzing functions in four ways (from tables of values, from function definitions, from written descriptions, and from graphs), your graphing calculator will be indispensible. We will be investigating functions, derivatives, and anti-derivatives using the graphing calculator to illustrate and discover characteristics of these three forms of a function as well as other appropriate calculus concepts (such as “limit”). Therefore you will learn and be required to use the following graphing calculator techniques:

1. find roots of functions and points of intersection
2. graph and interpret derivatives of functions (using “nderiv”)
3. graph and interpret anti-derivatives of functions (using “fnint”)
4. set appropriate windows
5. construct tables of values on home screen and in table mode
6. evaluate functions(including derivatives and anti-derivatives) using y-vars
7. evaluate sums for sequences (including Riemann sums and trapezoidal sums to approximate a definite integral numerically)

For example we will graph piece-wise functions so that we can support our work with calculus theorems regarding intervals of increase, intervals of decrease, concavity, points of non-differentiability(including discontinuities as well as corner points), and horizontal points. We will use the graphing calculator to graph the derivative of this piecewise function in order to interpret the results before finding the derivative of the function using appropriate rules for differentiation. Emphasis will be placed on making connections between the graphs of function and their behavior obtained by analyzing the functions using calculus techniques. You will be required to construct the possible graphs of f, f’, and f’’ given any of the three. As a class activity, we will use f(x) = sin(x) to illustrate this relationship by letting f(x) = sin(x) represent one of these three function forms. You will learn calculus this year and hopefully be challenged, yet enjoy, discovering new mathematical procedures and concepts while you gain an understanding of the connections of calculus to all of the mathematics you have learned in your previous math classes. **Prepare to be awed and amazed by the topics of calculus.**