



## Radnor High School Course Syllabus

Created 7/8/2013

### Keystone Mathematics 1509

<b>Credits:</b>	<b>0.5</b>	<b>Grades:</b>	<b>9-12</b>
<b>Weighted:</b>	<b>none</b>	<b>Prerequisite:</b>	<b>Keystone Exam score of Basic or Below Basic</b>
<b>Length:</b>	<b>full-year</b>		
<b>Format:</b>	<b>meets on alternate-day cycle</b>		

#### *Overall Description of Course*

This is a direct instruction course designed to reteach Algebra 1 concepts required by PDE for the Keystone Exam. It is a state-mandated remediation course for students who scored at the Basic or Below Basic level on the Keystone Exam. Topics covered include all state-required anchors listed on the PDE website for Algebra 1. This course meets all Keystone assessment anchors and PA common core standards.

#### Unit 1: Operations with Real #s and Expressions (part 1)

- Number line, number systems
- Radicals
- Greatest common factor (GCF), least common multiple (LCM)
- Exponents, roots, absolute value
- Simplifying expressions (monomials)

#### Unit 2: Operations with Real #s and Expressions (part 2)

- Estimation
- Polynomial expressions
- Factoring algebraic expressions
- Simplifying rational expressions

#### Unit 3: Linear Equations

- Properties while solving linear equations with one variable
- Word problems (set up one-variable equations)
- Writing linear functions in two variables from word problems

#### Unit 4: Linear Inequalities

- Linear inequalities in one variable
- Compound inequalities
- Absolute value inequalities in one variable

#### Unit 5: Functions

- Identifying and representing patterns
- Functions and relations
- Domain and range

#### Unit 6: Linear Functions

- Slopes, rates of change
- Point slope form
- Standard form

- Slope intercept form
- Line of best fit
- Predictions of line of best fit

Unit 7: Systems of Linear Equations/Inequalities

- Solving systems of linear equations
- Graphing linear inequalities in 2 variables
- Graphing systems of linear inequalities

Unit 8: Data Analysis

- Central tendency and dispersion
- Box-and-whisker, stem/leaf plots
- Predictions from data
- Probability of compound events

### *Common Core Standards*

**CC.2.1.8.E.1** Distinguish between rational and irrational numbers using their properties.

**CC.2.1.8.E.4** Estimate irrational numbers by comparing them to rational numbers.

**CC.2.1.HS.F.1** Apply and extend the properties of exponents to solve problems with rational exponents.

**CC.2.1.HS.F.2** Apply properties of rational and irrational numbers to solve real-world or mathematical problems.

**CC.2.1.6.E.3** Develop and/or apply number theory concepts to find common factors and multiples.

**CC.2.2.8.B.1** Apply concepts of radicals and integer exponents to generate equivalent expressions.

**CC.2.2.7.B.3** Model and solve real-world and mathematical problems by using and connecting numerical, algebraic, and/or graphical representations.

**CC.2.2.HS.D.9** Use reasoning to solve equations and justify the solution method.

**CC.2.2.HS.D.1** Interpret the structure of expressions to represent a quantity in terms of its context.

**CC.2.2.HS.D.2** Write expressions in equivalent forms to solve problems.

**CC.2.2.HS.D.3** Extend the knowledge of arithmetic operations and apply to polynomials.

**CC.2.2.HS.D.5** Use polynomial identities to solve problems.

**CC.2.2.HS.D.6** Extend the knowledge of rational functions to rewrite in equivalent forms.

**CC.2.1.HS.F.3** Apply quantitative reasoning to choose and interpret units and scales in formulas, graphs, and data displays.

**CC.2.1.HS.F.4** Use units as a way to understand problems and to guide the solution of multi-step problems.

**CC.2.1.HS.F.5** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

**CC.2.2.8.B.3** Analyze and solve linear equations and pairs of simultaneous linear equations.

**CC.2.2.8.C.1** Define, evaluate, and compare functions.

**CC.2.2.8.C.2** Use concepts of functions to model relationships between quantities.

**CC.2.2.HS.C.3** Write functions or sequences that model relationships between two quantities.

**CC.2.2.HS.D.7** Create and graph equations or inequalities to describe numbers or relationships.

**CC.2.2.HS.D.8** Apply inverse operations to solve equations or formulas for a given variable.

**CC.2.2.HS.D.9** Use reasoning to solve equations and justify the solution method.

**CC.2.2.HS.D.10** Represent, solve, and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically.

**CC.2.2.HS.C.1** Use the concept and notation of functions to interpret and apply them in terms of their context.

**CC.2.2.HS.C.2** Graph and analyze functions and use their properties to make connections between the different representations.

**CC.2.4.HS.B.2** Summarize, represent, and interpret data on two categorical and quantitative variables.

**CC.2.2.8.B.2** Understand the connections between proportional relationships, lines, and linear equations.

**CC.2.2.HS.C.4** Interpret the effects transformations have on functions and find the inverses of functions.

**CC.2.2.HS.C.6** Interpret functions in terms of the situations they model.

**CC.2.2.HS.C.5** Construct and compare linear, quadratic, and exponential models to solve problems.

**CC.2.4.HS.B.1** Summarize, represent, and interpret data on a single count or measurement variable.

**CC.2.4.8.B.1** Analyze and/or interpret bivariate data displayed in multiple representations.

**CC.2.4.HS.B.3** Analyze linear models to make interpretations based on the data.

**CC.2.4.HS.B.5** Make inferences and justify conclusions based on sample surveys, experiments, and observational studies.

**CC.2.4.7.B.3** Investigate chance processes and develop, use, and evaluate probability models.

**CC.2.4.HS.B.4** Recognize and evaluate random processes underlying statistical experiments.

**CC.2.4.HS.B.7** Apply the rules of probability to compute probabilities of compound events in a uniform probability model.

### *Keystone Connections:*

**A1.1.1.1.1** Compare and/or order any real numbers. Note: Rational and irrational may be mixed.

**A1.1.1.1.2** Simplify square roots (e.g.,  $\sqrt{24} = 2\sqrt{6}$ ).

**A1.1.1.2.1** Find the Greatest Common Factor (GCF) and/or the Least Common Multiple (LCM) for sets of monomials.

**A1.1.1.3.1** Simplify/evaluate expressions involving properties/laws of exponents, roots, and/or absolute values to solve problems. Note: Exponents should be integers from  $-10$  to  $10$ .

**A1.1.1.4.1** Use estimation to solve problems.

**A1.1.1.5.1** Add, subtract, and/or multiply polynomial expressions (express answers in simplest form). Note: Nothing larger than a binomial multiplied by a trinomial.

**A1.1.1.5.2** Factor algebraic expressions, including difference of squares and trinomials. Note: Trinomials are limited to the form  $ax^2 + bx + c$  where  $a$  is equal to  $1$  after factoring out all monomial factors.

**A1.1.1.5.3** Simplify/reduce a rational algebraic expression.

**A1.1.2.1.1** Write, solve, and/or apply a linear equation (including problem situations).

**A1.1.2.1.2** Use and/or identify an algebraic property to justify any step in an equation-solving process. Note: Linear equations only.

**A1.1.2.1.3** Interpret solutions to problems in the context of the problem situation. Note: Linear equations only.

**A1.1.2.2.1** Write and/or solve a system of linear equations (including problem situations) using graphing, substitution, and/or elimination. Note: Limit systems to two linear equations.

**A1.1.2.2.2** Interpret solutions to problems in the context of the problem situation. Note: Limit systems to two linear equations.

**A1.1.3.1.1** Write or solve compound inequalities and/or graph their solution sets on a number line (may include absolute value inequalities).

**A1.1.3.1.2** Identify or graph the solution set to a linear inequality on a number line.

**A1.1.3.1.3** Interpret solutions to problems in the context of the problem situation.

Note: Linear inequalities only.

**A1.1.3.2.1** Write and/or solve a system of linear inequalities using graphing.

Note: Limit systems to two linear inequalities.

**A1.1.3.2.2** Interpret solutions to problems in the context of the problem situation.

Note: Limit systems to two linear inequalities.

**A1.2.1.1.1** Analyze a set of data for the existence of a pattern and represent the pattern algebraically and/or graphically.

**A1.2.1.1.2** Determine whether a relation is a function, given a set of points or a graph.

**A1.2.1.1.3** Identify the domain or range of a relation (may be presented as ordered pairs, a graph, or a table).

**A1.2.1.2.1** Create, interpret, and/or use the equation, graph, or table of a linear function.

**A1.2.1.2.2** Translate from one representation of a linear function to another (i.e., graph, table, and equation).

**A1.2.2.1.1** Identify, describe, and/or use constant rates of change.

**A1.2.2.1.2** Apply the concept of linear rate of change (slope) to solve problems.

**A1.2.2.1.3** Write or identify a linear equation when given

- the graph of the line,
- two points on the line, or
- the slope and a point on the line.

Note: Linear equation may be in point-slope, standard, and/or slope-intercept form.

**A1.2.2.1.4** Determine the slope and/or  $y$ -intercept represented by a linear equation or graph.

**A1.2.2.2.1** Draw, identify, find, and/or write an equation for a line of best fit for a scatter plot.

**A1.2.3.1.1** Calculate and/or interpret the range, quartiles, and interquartile range of data.

**A1.2.3.2.1** Estimate or calculate to make predictions based on a circle, line, bar graph, measure of central tendency, or other representation.

**A1.2.3.2.2** Analyze data, make predictions, and/or answer questions based on displayed data (box-and-whisker plots, stem-and-leaf plots, scatter plots, measures of central tendency, or other representations).

**A1.2.3.2.3** Make predictions using the equations or graphs of best-fit lines of scatter plots.

**A1.2.3.3.1** Find probabilities for compound events (e.g., find probability of red and blue, find probability of red or blue) and represent as a fraction, decimal, or percent.

### *Student Objectives:*

To develop algebraic skills and concepts which enhance problem solving ability.

To reason and communicate mathematically.

To master Algebra 1 concepts to a Proficient level.

### *Materials & Texts*

Keystone Finish Line: Algebra 1. Continental Press Co., Inc. 2013 (teacher resource)

TI-84 graphing calculator

Homework, classwork and assessments modeled from eligible content of Keystone Anchors from PDE website.

## *Activities, Assignments, & Assessments*

### **ASSIGNMENTS**

Homework will be assigned on a daily basis, and will be graded.

### **ASSESSMENTS**

Grades will be based primarily on unit tests. CDTs will be administered throughout the year to show growth.

## *Terminology*

**Absolute Value** - A number's distance from zero on the number line. It is written  $|a|$  and is read "the absolute value of a."

**Additive Inverse** - The opposite of a number (i.e., for any number  $a$ , the additive inverse is  $-a$ ).

**Arithmetic Sequence** - An ordered list of numbers that increases or decreases at a constant rate (i.e., the difference between numbers remains the same).

**Asymptote** - A straight line to which the curve of a graph comes closer and closer. The distance between the curve and the asymptote approaches zero as they tend to infinity. The asymptote is denoted by a dashed line on a graph. The most common asymptotes are horizontal and vertical.

**Bar Graph** - A graph that shows a set of frequencies using bars of equal width, but heights that are proportional to the frequencies. It is used to summarize discrete data.

**Binomial** - A polynomial with two unlike terms (e.g.,  $3x + 4y$  or  $3 - 4b$ ). Each term is a monomial, and the monomials are joined by an addition symbol (+) or a subtraction symbol (-). It is considered an algebraic expression.

**Box-and-Whisker Plot** - A graphic method for showing a summary and distribution of data using median, quartiles, and extremes (i.e., minimum and maximum) of data. This shows how far apart and how evenly data is distributed. It is helpful when a visual is needed to see if a distribution is skewed or if there are any outliers.

**Circle Graph (or Pie Chart)** - A circular diagram using different-sized sectors of a circle whose angles at the center are proportional to the frequency. Sectors can be visually compared to show information (e.g., statistical data). Sectors resemble slices of a pie.

**Coefficient** - The number, usually a constant, that is multiplied by a variable in a term (e.g., 35 is the coefficient of  $35x^2y$ ); the absence of a coefficient is the same as a 1 being present (e.g.,  $x$  is the same as  $1x$ ).

**Combination** - An unordered arrangement, listing or selection of objects (e.g., two-letter combinations of the three letters X, Y, and Z would be XY, XZ, and YZ; XY is the same as YX and is not counted as a different combination). A combination is similar to, but not the same as, a permutation.

**Common Logarithm** - A logarithm with base 10. It is written  $\log x$ . The common logarithm is the power of 10 necessary to equal a given number (i.e.,  $\log x = y$  is equivalent to  $10^y = x$ ).

**Complex Number** - The sum or difference of a real number and an imaginary number. It is written in the form  $a + bi$ , where  $a$  and  $b$  are real numbers and  $i$  is the imaginary unit.

**Composite Number** - Any natural number with more than two factors (e.g., 6 is a composite number since it has four factors: 1, 2, 3, and 6). A composite number is not a prime number.

**Compound (or Combined) Event** - An event that is made up of two or more simple events, such as the flipping of two or more coins.

**Compound Inequality** - When two or more inequalities are taken together and written with the

inequalities connected by the words and or or (e.g.,  $x > 6$  and  $x < 12$ , which can also be written as  $6 < x < 12$ ).

**Constant** - A term or expression with no variable in it. It has the same value all the time.

**Coordinate Plane** - A plane formed by perpendicular number lines. The horizontal number line is the x-axis, and the vertical number line is the y-axis. The point where the axes meet is called the origin.

**Cube Root** - One of three equal factors (roots) of a number or expression; a radical expression with a degree of 3.

**Curve of Best Fit (for a Scatter Plot)** - See line or curve of best fit (for a scatter plot).

**Degree (of a Polynomial)** - The value of the greatest exponent in a polynomial.

**Dependent Events** - Two or more events in which the outcome of one event affects or influences the outcome of the other event(s).

**Dependent Variable** - The output number or variable in a relation or function that depends upon another variable, called the independent variable, or input number (e.g., in the equation  $y = 2x + 4$ ,  $y$  is the dependent variable since its value depends on the value of  $x$ ). It is the variable for which an equation is solved. Its values make up the range of the relation or function.

**Domain (of a Relation or Function)** - The set of all possible values of the independent variable on which a function or relation is allowed to operate. Also, the first numbers in the ordered pairs of a relation; the values of the x-coordinates in  $(x, y)$ .

**Elimination Method** - See linear combination.

**Equation** - A mathematical statement or sentence that says one mathematical expression or quantity is equal to another (e.g.,  $x + 5 = y - 7$ ). An equation will always contain an equal sign (=).

**Estimation** - Strategy An approximation based on a judgment; may include determining approximate values, establishing the reasonableness of answers, assessing the amount of error resulting from estimation, and/or determining if an error is within acceptable limits.

**Exponent** - The power to which a number or expression is raised. When the exponent is a fraction, the number or expression can be rewritten with a radical sign. See also positive exponent and negative exponent.

**Exponential Equation** - An equation with variables in its exponents (e.g.,  $4^x = 50$ ). It can be solved by taking logarithms of both sides.

**Exponential Expression** - An expression in which the variable occurs in the exponent (such as  $4^x$  rather than  $x^4$ ). Often it occurs when a quantity changes by the same factor for each unit of time (e.g., "doubles every year" or "decreases 2% each month").

**Exponential Function (or Model)** - A function whose general equation is  $y = a \cdot b^x$  where  $a$  and  $b$  are constants.

**Exponential Growth/Decay** - A situation where a quantity increases or decreases exponentially by the same factor over time; it is used for such phenomena as inflation, population growth, radioactivity or depreciation.

**Expression** - A mathematical phrase that includes operations, numbers, and/or variables (e.g.,  $2x + 3y$  is an algebraic expression,  $13.4 - 4.7$  is a numeric expression). An expression does not contain an equal sign (=) or any type of inequality sign.

**Factor (noun)** - The number or expression that is multiplied by another to get a product (e.g., 6 is a factor of 30, and  $6x$  is a factor of  $42x^2$ ).

**Factor (verb)** - To express or write a number, monomial, or polynomial as a product of two or more factors.

**Factor a Monomial** - To express a monomial as the product of two or more monomials.

**Factor a Polynomial** - To express a polynomial as the product of monomials and/or polynomials (e.g., factoring the polynomial  $x^2 + x - 12$  results in the product  $(x - 3)(x + 4)$ ).

**Frequency** - How often something occurs (i.e., the number of times an item, number, or event

happens in a set of data).

**Function** - A relation in which each value of an independent variable is associated with a unique value of a dependent variable (e.g., one element of the domain is paired with one and only one element of the range). It is a mapping which involves either a one-to-one correspondence or a many-to-one correspondence, but not a one-to-many correspondence.

**Fundamental Counting Principle** - A way to calculate all of the possible combinations of a given number of events. It states that if there are  $x$  different ways of doing one thing and  $y$  different ways of doing another thing, then there are  $xy$  different ways of doing both things. It uses the multiplication rule.

**Geometric Sequence** - An ordered list of numbers that has the same ratio between consecutive terms (e.g., 1, 7, 49, 343, ... is a geometric sequence that has a ratio of  $7/1$  between consecutive terms; each term after the first term can be found by multiplying the previous term by a constant, in this case the number 7 or  $7/1$ ).

**Greatest Common Factor (GCF)** - The largest factor that two or more numbers or algebraic terms have in common. In some cases the GCF may be 1 or one of the actual numbers (e.g., the GCF of  $18x^3$  and  $24x^5$  is  $6x^3$ ).

**Imaginary Number** - The square root of a negative number, or the opposite of the square root of a negative number. It is written in the form  $bi$ , where  $b$  is a real number and  $i$  is the imaginary root.

**Independent Event(s)** - Two or more events in which the outcome of one event does not affect the outcome of the other event(s) (e.g., tossing a coin and rolling a number cube are independent events). The probability of two independent events is equal to the product of the probabilities of the two individual events).

**Independent Variable** - The input number or variable in a relation or function whose value is subject to choice. It is not dependent upon any other values. It is usually the  $x$ -value or the  $x$  in  $f(x)$ . It is graphed on the  $x$ -axis. Its values make up the domain of the relation or function.

**Inequality** - A mathematical sentence that contains an inequality symbol (i.e.,  $>$ ,  $<$ ,  $\geq$ ,  $\leq$ , or  $\neq$ ). It compares two quantities. The symbol  $>$  means greater than, the symbol  $<$  means less than, the symbol  $\geq$  means greater than or equal to, the symbol  $\leq$  means less than or equal to, and the symbol  $\neq$  means not equal to.

**Integer** - A natural number, the additive inverse of a natural number, or zero. Any number from the set of numbers represented by  $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$ .

**Interquartile Range (of Data)** - The difference between the first (lower) and third (upper) quartile. It represents the spread of the middle 50% of a set of data.

**Inverse (of a Relation)** - A relation in which the coordinates in each ordered pair are switched from a given relation. The point  $(x, y)$  becomes  $(y, x)$ , so  $(3, 8)$  would become  $(8, 3)$ .

**Irrational Number** - A real number that cannot be written as a simple fraction (i.e., the ratio of two integers). It is a nonterminating (infinite) and non-repeating decimal. The square root of any prime number is irrational, as are  $\pi$  and  $e$ .

**Least (or Lowest) Common Multiple (LCM)** - The smallest number or expression that is a common multiple of two or more numbers or algebraic terms, other than zero.

**Like Terms** - Monomials that contain the same variables and corresponding powers and/or roots. Only the coefficients can be different (e.g.,  $4x^3$  and  $12x^3$ ). Like terms can be added or subtracted.

**Line Graph** - A graph that uses a line or line segments to connect data points, plotted on a coordinate plane, usually to show trends or changes in data over time. More broadly, a graph to represent the relationship between two continuous variables.

**Line or Curve of Best Fit (for a Scatter Plot)** - A line or curve drawn on a scatter plot to best estimate the relationship between two sets of data. It describes the trend of the data. Different measures are possible to describe the best fit. The most common is a line or curve that minimizes the sum of the

squares of the errors (vertical distances) from the data points to the line. The line of best fit is a subset of the curve of best fit.

**Linear Combination** - A method by which a system of linear equations can be solved. It uses addition or subtraction in combination with multiplication or division to eliminate one of the variables in order to solve for the other variable.

**Linear Equation** - An equation for which the graph is a straight line (i.e., a polynomial equation of the first degree of the form  $Ax + By = C$ , where  $A$ ,  $B$ , and  $C$  are real numbers and where  $A$  and  $B$  are not both zero; an equation in which the variables are not multiplied by one another or raised to any power other than 1).

**Linear Function** - A function for which the graph is a non-vertical straight line. It is a first degree polynomial of the common form  $f(x) = mx + b$ , where  $m$  and  $b$  are constants and  $x$  is a real variable. The constant  $m$  is called the slope and  $b$  is called the  $y$ -intercept. It has a constant rate of change.

**Linear Inequality** - The relation of two expressions using the symbols  $<$ ,  $>$ ,  $\leq$ ,  $\geq$ , or  $\neq$  and whose boundary is a straight line. The line divides the coordinate plane into two parts. If the inequality is either  $\leq$  or  $\geq$ , then the boundary is solid. If the inequality is either  $<$  or  $>$ , then the boundary is dashed. If the inequality is  $\neq$ , then the solution contains everything except for the boundary.

**Logarithm** - The exponent required to produce a given number (e.g., since 2 raised to a power of 5 is 32, the logarithm base 2 of 32 is 5; this is written as  $\log_2 32 = 5$ ). Two frequently used bases are 10 (common logarithm) and  $e$  (natural logarithm). When a logarithm is written without a base, it is understood to be base 10.

**Logarithmic Equation** - An equation which contains a logarithm of a variable or number. Sometimes it is solved by rewriting the equation in exponential form and solving for the variable (e.g.,  $\log_2 32 = 5$  is the same as  $2^5 = 32$ ). It is an inverse function of the exponential function.

**Mapping** - The matching or pairing of one set of numbers to another by use of a rule. A number in the domain is matched or paired with a number in the range (or a relation or function). It may be a one-to-one correspondence, a one-to-many correspondence, or a many-to-one correspondence.

**Maximum Value (of a Graph)** - The value of the dependent variable for the highest point on the graph of a curve.

**Mean** - A measure of central tendency that is calculated by adding all the values of a set of data and dividing that sum by the total number of values. Unlike median, the mean is sensitive to outlier values. It is also called "arithmetic mean" or "average".

**Measure of Central Tendency** - A measure of location of the middle (center) of a distribution of a set of data (i.e., how data clusters). The three most common measures of central tendency are mean, median, and mode.

**Measure of Dispersion** - A measure of the way in which the distribution of a set of data is spread out. In general the more spread out a distribution is, the larger the measure of dispersion. Range and interquartile range are two measures of dispersion.

**Median** - A measure of central tendency that is the middle value in an ordered set of data or the average of the two middle values when the set has two middle values (occurs when the set of data has an even number of data points). It is the value halfway through the ordered set of data, below and above which there is an equal number of data values. It is generally a good descriptive measure for skewed data or data with outliers.

**Minimum Value (of a Graph)** - The value of the dependent variable for the lowest point on the graph of a curve.

**Mode** - A measure of central tendency that is the value or values that occur(s) most often in a set of data. A set of data can have one mode, more than one mode, or no mode.

**Monomial** - A polynomial with only one term; it contains no addition or subtraction. It can be a number, a variable, or a product of numbers and/or more variables.

**Multiplicative Inverse** - The reciprocal of a number.

**Mutually Exclusive Events** - Two events that cannot occur at the same time (i.e., events that have no outcomes in common). If two events A and B are mutually exclusive, then the probability of A or B occurring is the sum of their individual probabilities:  $P(A \cup B) = P(A) + P(B)$ . Also defined as when the intersection of two sets is empty, written as  $A \cap B = \emptyset$ .

**Natural Logarithm** - A logarithm with base e. It is written  $\ln x$ . The natural logarithm is the power of e necessary to equal a given number (i.e.,  $\ln x = y$  is equivalent to  $e^y = x$ ). The constant e is an irrational number whose value is approximately 2.71828....

**Natural Number** - A counting number. A number representing a positive, whole amount. Any number from the set of numbers represented by  $\{1, 2, 3, \dots\}$ . Sometimes, it is referred to as a "positive integer".

**Negative Exponent** - An exponent that indicates a reciprocal that has to be taken before the exponent can be applied. It is used in scientific notation for numbers between  $-1$  and  $1$ .

**Number Line** - A graduated straight line that represents the set of all real numbers in order. Typically, it is marked showing integer values.

**Odds** - A comparison, in ratio form (as a fraction or with a colon), of outcomes. "Odds in favor" (or simply "odds") is the ratio of favorable outcomes to unfavorable outcomes (e.g., the odds in favor of picking a red hat when there are 3 red hats and 5 non-red hats is 3:5). "Odds against" is the ratio of unfavorable outcomes to favorable outcomes (e.g., the odds against picking a red hat when there are 3 red hats and 5 non-red hats is 5:3).

**Order of Operations** - Rules describing what order to use in evaluating expressions:

- (1) Perform operations in grouping symbols (parentheses and brackets),
- (2) Evaluate exponential expressions and radical expressions from left to right,
- (3) Multiply or divide from left to right,
- (4) Add or subtract from left to right.

**Ordered Pair** - A pair of numbers used to locate a point on a coordinate plane, or the solution of an equation in two variables. The first number tells how far to move horizontally, and the second number tells how far to move vertically; written in the form (x-coordinate, y-coordinate). Order matters: the point (x, y) is not the same as (y, x).

**Origin** - The point (0, 0) on a coordinate plane. It is the point of intersection for the x-axis and the y-axis.

**Outlier** - A value that is much greater or much less than the rest of the data. It is different in some way from the general pattern of data. It directly stands out from the rest of the data. Sometimes it is referred to as any data point more than 1.5 interquartile ranges greater than the upper (third) quartile or less than the lower (first) quartile.

**Pattern (or Sequence)** - A set of numbers arranged in order (or in a sequence). The numbers and their arrangement are determined by a rule, including repetition and growth/decay rules. See arithmetic sequence and geometric sequence.

**Perfect Square** - A number whose square root is a whole number (e.g., 25 is a perfect square since  $25 = 5^2$ ). A perfect square can be found by raising a whole number to the second power (e.g.,  $5^2 = 25$ ).

**Permutation** - An ordered arrangement of objects from a given set in which the order of the objects is significant (e.g., two-letter permutations of the three letters X, Y, and Z would be XY, YX, XZ, ZX, YZ, and ZY). A permutation is similar to, but not the same as, a combination.

**Point-Slope Form (of a Linear Equation)** - An equation of a straight, non-vertical line written in the form  $y - y_1 = m(x - x_1)$ , where m is the slope of the line and  $(x_1, y_1)$  is a given point on the line. This can also be written as  $y = m(x - h) + k$  where m is the slope of the line, and (h, k) is a given point on the line.

**Polynomial** - An algebraic expression that is a monomial or the sum or difference of two or more

monomials (e.g.,  $6a$  or  $5a^2 + 3a - 13$  where the exponents are natural numbers).

**Polynomial Function** - A function of the form  $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$ , where  $a_n \neq 0$  and natural number  $n$  is the degree of the polynomial.

**Positive Exponent** - Indicates how many times a base number is multiplied by itself. In the expression  $x^n$ ,  $n$  is the positive exponent, and  $x$  is the base number (e.g.,  $2^3 = 2 \cdot 2 \cdot 2$ ).

**Power** - The value of the exponent in a term. The expression  $a^n$  is read "a to the power of n." To raise a number,  $a$ , to the power of another whole number,  $n$ , is to multiply  $a$  by itself  $n$  times (e.g., the number  $4^3$  is read "four to the third power" and represents  $4 \cdot 4 \cdot 4$ ).

**Power of a Power** - An expression of the form  $(a^m)^n$ . It can be found by multiplying the exponents (e.g.,  $(2^3)^4 = 2^{3 \cdot 4} = 2^{12} = 4,096$ ).

**Powers of Products** - An expression of the form  $a^m \cdot a^n$ . It can be found by adding the exponents when multiplying powers that have the same base (e.g.,  $2^3 \cdot 2^4 = 2^{3+4} = 2^7 = 128$ ).

**Prime Number** - Any natural number with exactly two factors, 1 and itself (e.g., 3 is a prime number since it has only two factors: 1 and 3). [Note: Since 1 has only one factor, itself, it is not a prime number.] A prime number is not a composite number.

**Probability** - A number from 0 to 1 (or 0% to 100%) that indicates how likely an event is to happen. A very unlikely event has a probability near 0 (or 0%) while a very likely event has a probability near 1 (or 100%). It is written as a ratio (fraction, decimal, or equivalent percent). The number of ways an event could happen (favorable outcomes) is placed over the total number of events (total possible outcomes) that could happen. A probability of 0 means it is impossible, and a probability of 1 means it is certain.

**Probability of a Compound (or Combined) Event** - There are two types:

1. The union of two events  $A$  and  $B$ , which is the probability of  $A$  or  $B$  occurring. This is represented as  $P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B)$ .

2. The intersection of two events  $A$  and  $B$ , which is the probability of  $A$  and  $B$  occurring. This is represented as  $P(A \cap B) = P(A) \cdot P(B)$ .

**Quadrants** - The four regions of a coordinate plane that are separated by the  $x$ -axis and the  $y$ -axis, as shown below.

(1) The first quadrant (Quadrant I) contains all the points with positive  $x$  and positive  $y$  coordinates (e.g.,  $(3, 4)$ ).

(2) The second quadrant (Quadrant II) contains all the points with negative  $x$  and positive  $y$  coordinates (e.g.,  $(-3, 4)$ ).

(3) The third quadrant (Quadrant III) contains all the points with negative  $x$  and negative  $y$  coordinates (e.g.,  $(-3, -4)$ ).

(4) The fourth quadrant (Quadrant IV) contains all the points with positive  $x$  and negative  $y$  coordinates (e.g.,  $(3, -4)$ ).

**Quadratic Equation** - An equation that can be written in the standard form  $ax^2 + bx + c = 0$ , where  $a$ ,  $b$ , and  $c$  are real numbers and  $a$  does not equal zero. The highest power of the variable is 2. It has, at most, two solutions. The graph is a parabola.

**Quadratic Formula** - The solutions or roots of a quadratic equation in the form  $ax^2 + bx + c = 0$ , where

$a \neq 0$ , are given by the formula  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

**Quadratic Function** - A function that can be expressed in the form  $f(x) = ax^2 + bx + c$ , where  $a \neq 0$  and the highest power of the variable is 2. The graph is a parabola.

**Quartile** - One of three values that divides a set of data into four equal parts:

1. Median divides a set of data into two equal parts.

2. Lower quartile ( $25^{\text{th}}$  percentile) is the median of the lower half of the data.

**3. Upper quartile (75<sup>th</sup> percentile) is the median of the upper half of the data.**

**Radical Expression** - An expression containing a radical symbol ( $\sqrt[n]{a}$ ). The expression or number inside the radical (a) is called the radicand, and the number appearing above the radical (n) is the degree. The degree is always a positive integer. When a radical is written without a degree, it is understood to be a degree of 2 and is read as “the square root of a.” When the degree is 3, it is read as “the cube root of a.” For any other degree, the expression

$\sqrt[n]{a}$  is read as “the nth root of a.” When the degree is an even number, the radical expression is assumed to be the principal (positive) root.

**Range (of a Relation or Function)** - The set of all possible values for the output (dependent variable) of a function or relation; the set of second numbers in the ordered pairs of a function or relation; the values of the y-coordinates in (x, y).

**Range (of Data)** - In statistics, a measure of dispersion that is the difference between the greatest value (maximum value) and the least value (minimum value) in a set of data.

**Rate** - A ratio that compares two quantities having different units.

**Rate (of Change)** - The amount a quantity changes over time (e.g., 3.2 cm per year). Also the amount a function’s output changes (increases or decreases) for each unit of change in the input. See slope.

**Rate (of Interest)** - The percent by which a monetary account accrues interest. It is most common for the rate of interest to be measured on an annual basis (e.g., 4.5% per year), even if the interest is compounded periodically (i.e., more frequently than once per year).

**Ratio** - A comparison of two numbers, quantities or expressions by division. It is often written as a fraction, but not always (e.g.,  $\frac{2}{3}$ , 2:3, 2 to 3,  $2 \div 3$  are all the same ratios).

**Rational Expression** - An expression that can be written as a polynomial divided by a polynomial, defined only when the latter is not equal to zero.

**Rational Number** - Any number that can be written in the form  $\frac{a}{b}$  where a is any integer and b is any integer except zero. All repeating decimal and terminating decimal numbers are rational numbers.

**Real Number** - The combined set of rational and irrational numbers. All numbers on the number line. Not an imaginary number.

**Regression Curve** - The line or curve of best fit that represents the least deviation from the points in a scatter plot of data. Most commonly it is linear and uses a “least squares” method.

**Relation** - A set of pairs of values (e.g.,  $\{(1, 2), (2, 3), (3, 2)\}$ ). The first value in each pair is the input (independent value), and the second value in the pair is the output (dependent value). In a relation, neither the input values nor the output values need to be unique.

**Repeating Decimal** - A decimal with one or more digits that repeats endlessly (e.g., 0.666..., 0.727272..., 0.08333...). To indicate the repetition, a bar may be written above the repeated digits. All repeating decimals are rational numbers.

**Rise** - The vertical (up and down) change or difference between any two points on a line on a coordinate plane (i.e., for points  $(x_1, y_1)$  and  $(x_2, y_2)$ , the rise is  $y_2 - y_1$ ). See slope.

**Run** - The horizontal (left and right) change or difference between any two points on a line on a coordinate plane (i.e., for points  $(x_1, y_1)$  and  $(x_2, y_2)$ , the run is  $x_2 - x_1$ ). See slope.

**Scatter Plot** - A graph that shows the “general” relationship between two sets of data. For each point that is being plotted there are two separate pieces of data. It shows how one variable is affected by another.

**Simple Event** - When an event consists of a single outcome (e.g., rolling a number cube).

**Simplest Form (of an Expression)** - When all like terms are combined (e.g.,  $8x + 2(6x - 22)$  becomes  $20x - 44$  when in simplest form). The form which no longer contains any like terms, parentheses, or reducible fractions.

**Simplify** - To write an expression in its simplest form (i.e., remove any unnecessary terms, usually by

combining several or many terms into fewer terms or by cancelling terms).

**Slope (of a Line)** - A rate of change. The measurement of the steepness, incline, or grade of a line from left to right. It is the ratio of vertical change to horizontal change. More specifically, it is the ratio of the change in the y-coordinates (rise) to the corresponding change in the x-coordinates (run) when moving from one point to another along a line. It also indicates whether a line is tilted upward (positive slope) or downward (negative slope) and is written as

the letter m where  $m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$

**Slope-Intercept Form** - An equation of a straight, non-vertical line written in the form  $y = mx + b$ , where m is the slope and b is the y-intercept.

**Square Root** - One of two equal factors (roots) of a number or expression; a radical expression ( a ) with an understood degree of 2. The square root of a number or expression is assumed to be the principal (positive) root (e.g.,  $\sqrt{49x^2}$ ). The square root of a negative number results in an imaginary number (e.g.,  $\sqrt{-49} = 7i$ ).

**Standard Form (of a Linear Equation)** - An equation of a straight line written in the form  $Ax + By = C$ , where A, B, and C are real numbers and where A and B are not both zero. It includes variables on one side of the equation and a constant on the other side.

**Stem-and-Leaf Plot** - A visual way to display the shape of a distribution that shows groups of data arranged by place value; a way to show the frequency with which certain classes of data occur. The stem consists of a column of the larger place value(s); these numbers are not repeated. The leaves consist of the smallest place value (usually the ones place) of every piece of data; these numbers are arranged in numerical order in the row of the appropriate stem (e.g., the number 36 would be indicated by a leaf of 6 appearing in the same row as the stem of 3).

**Substitution** - The replacement of a term or variable in an expression or equation by another that has the same value in order to simplify or evaluate the expression or equation.

**System of Linear Equations** - A set of two or more linear equations with the same variables. The solution to a system of linear equations may be found by linear combination, substitution, or graphing. A system of two linear equations will either have one solution, infinitely many solutions, or no solutions.

**System of Linear Inequalities** - Two or more linear inequalities with the same variables. Some systems of inequalities may include equations as well as inequalities. The solution region may be closed or bounded because there are lines on all sides, while other solutions may be open or unbounded.

**Systems of Equations** - A set of two or more equations containing a set of common variables.

**Term** - A part of an algebraic expression. Terms are separated by either an addition symbol (+) or a subtraction symbol (-). It can be a number, a variable, or a product of a number and one or more variables (e.g., in the expression  $4x^2 + 6y$ ,  $4x^2$  and  $6y$  are both terms).

**Terminating Decimal** - A decimal with a finite number of digits. A decimal for which the division operation results in either repeating zeroes or repeating nines (e.g.,  $0.375000\dots = 0.375$ ,  $0.1999\dots = 0.2$ ). It is generally written to the last non-zero place value, but can also be written with additional zeroes in smaller place values as needed (e.g., 0.25 can also be written as 0.2500). All terminating decimals are rational numbers.

**Trinomial** - A polynomial with three unlike terms (e.g.,  $7a + 4b + 9c$ ). Each term is a monomial, and the monomials are joined by an addition symbol (+) or a subtraction symbol (-). It is considered an algebraic expression.

**Unit Rate** - A rate in which the second (independent) quantity of the ratio is 1 (e.g., 60 words per minute, \$4.50 per pound, 21 students per class).

**Variable** - A letter or symbol used to represent any one of a given set of numbers or other objects (e.g., in the equation  $y = x + 5$ , the  $y$  and  $x$  are variables). Since it can take on different values, it is the opposite of a constant.

**Whole Number** - A natural number or zero. Any number from the set of numbers represented by  $\{0, 1, 2, 3, \dots\}$ . Sometimes it is referred to as a “non-negative integer”.

**x-Axis** - The horizontal number line on a coordinate plane that intersects with a vertical number line, the  $y$ -axis; the line whose equation is  $y = 0$ . The  $x$ -axis contains all the points with a zero  $y$ -coordinate (e.g.,  $(5, 0)$ ).

**x-Intercept(s)** - The  $x$ -coordinate(s) of the point(s) at which the graph of an equation crosses the  $x$ -axis (i.e., the value(s) of the  $x$ -coordinate when  $y = 0$ ). The solution(s) or root(s) of an equation that is set equal to 0.

**y-Axis** - The vertical number line on a coordinate plane that intersects with a horizontal number line, the  $x$ -axis; the line whose equation is  $x = 0$ . The  $y$ -axis contains all the points with a zero  $x$ -coordinate (e.g.,  $(0, 7)$ ).

**y-Intercept(s)** - The  $y$ -coordinate(s) of the point(s) at which the graph of an equation crosses the  $y$ -axis (i.e., the value(s) of the  $y$ -coordinate when  $x = 0$ ). For a linear equation in slope-intercept form ( $y = mx + b$ ), it is indicated by  $b$ .

(Assessment Anchor & Eligible Content Glossary; PDE website, January 2013)

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