Differentiating the Curriculum for Elementary Gifted Mathematics Students

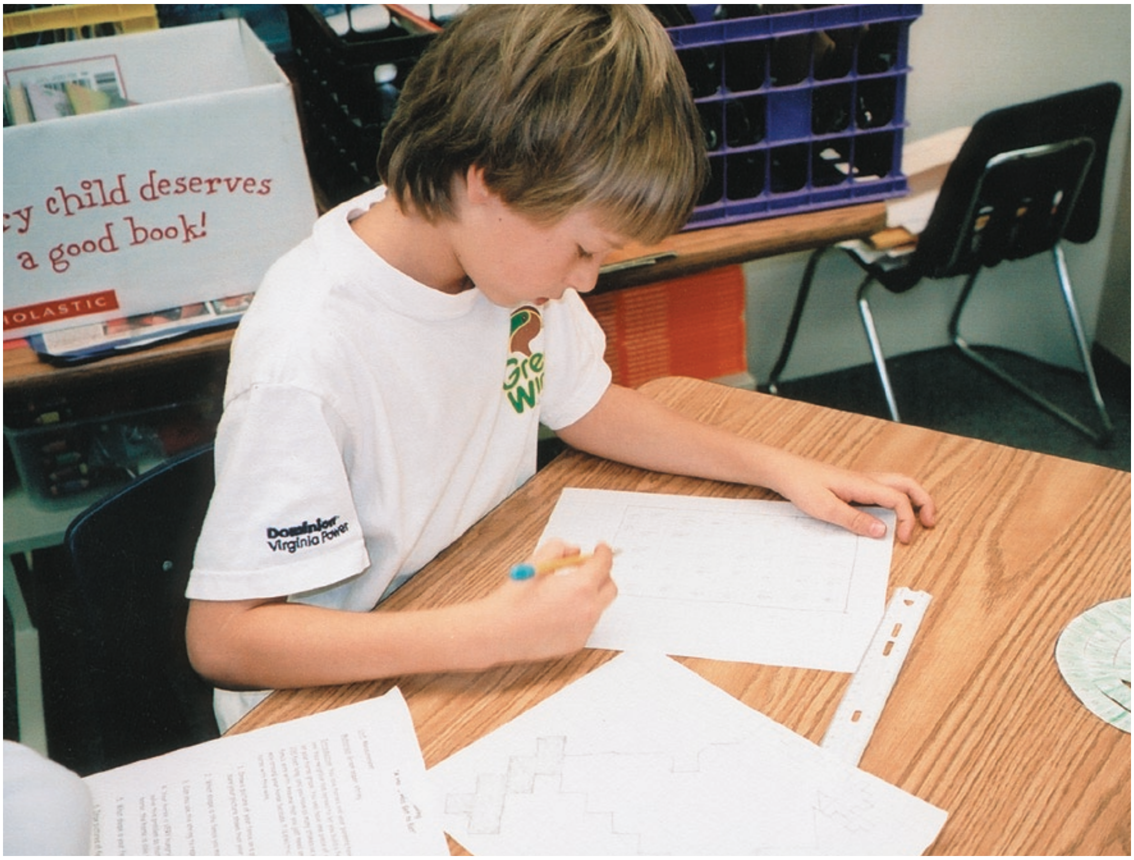
Carlos, a first grader, comes home from school one day looking upset. When Mom asks what is wrong, Carlos says that he got in trouble for working ahead in his mathematics workbook and that his teacher took it away from him. He had been asked not to work ahead because it would mean that he would not have anything to do at mathematics time when those pages were assigned. Carlos was disobedient, but Mom's heart goes out to him. He was just trying to fill the mathematics lesson time with something he enjoyed. It is not his fault that he already knows how to do the mathematics that will be taught several weeks in the future and that he can finish a 45-minute lesson's worth of curriculum in 10 minutes. Carlos is just a normal gifted mathematics student, but he is a real challenge for his classroom teacher.

By Michelle Muller Wilkins, Jesse L. M. Wilkins, and Tamra Oliver

Michelle Wilkins, miwilki1@vt.edu, volunteers regularly in her children's classrooms in Blacksburg, Virginia. She is interested in the ways activities help children learn. Jay Wilkins, wilkins@vt.edu, teaches elementary mathematics methods courses at Virginia Tech in Blacksburg. His research interests include quantitative literacy, educational opportunities, and the teaching and learning of probability and statistics. Tamra Oliver, toliver@mcps.org, is a consulting teacher for the gifted in the Montgomery County Public Schools in Blacksburg. Her interests include helping teachers and parents provide appropriate rigor and challenge for gifted students.

"Imagine a classroom, a school, or a school district where all students have access to high-quality, engaging mathematics instruction. There are ambitious expectations for all, with accommodation for those who need it."*—Principles and Standards for School Mathematics *(NCTM 2000, p. 3)

Gifted mathematics students need accommodation in the mathematics curriculum, but frequently they are not appropriately challenged. Often, gifted mathematics students are asked to work independently or help others with their work (Winebrenner 2001). Sometimes, these students are given extra worksheets to complete or more difficult problems of the same type (Galbraith 1998). Sometimes, as in the true story of Carlos in the opening vignette, gifted students are just asked to sit quietly in their seats for long periods while others in the class are finishing assigned work. Although there are clear benefits to helping others (Cohen 1982) and some learning may be gleaned by doing



extra work, gifted students are properly challenged when they are asked to move beyond computation into higher-order mathematical thinking processes such as applying computational skills to everyday problems, problem solving, problem posing, and creating new mathematics (Sheffield 2003).

This article presents a framework for creating a Mathematics Investigation Center (MIC) for gifted mathematics students. The goal of the framework is to make it easier for elementary teachers to provide challenging activities for students working above their grade level in mathematics without having to plan a separate lesson every day. The concept is to provide enrichment activities that generally fit the theme of the unit that the whole class is working on, instead of developing enrichment activities that are tailored to individual lessons. The activities focus on processes of mathematics rather than computation skills in an attempt to provide depth rather than breadth. Because gifted education is widely underfunded (DeLacy 2004) and many schools do not have aides or specialists who can regularly work with gifted children in the classroom, the students are expected to work semi-independently on MIC activities, and the teacher's role is to facilitate rather than instruct.

The Framework

Classroom setup

The Mathematics Investigation Center (MIC) for a given unit consisted of nine activities kept in separate folders in a box with any necessary manipulatives. Initially, there were nine activities because the Winebrenner format that we used to set up the center called for a 3 × 3 menu in tic-tac-toe style (see **fig. 1**). The activities remained available to the students for the duration of the unit. Students worked at the MIC during mathematics time once or twice a week at the teacher's discretion. The center was used in conjunction with informal curriculum compacting (Smutny 2001; Strip 2000). Students were asked to complete many of their regular assignments but were not asked to work at the MIC outside of the designated mathematics period so that gifted students would not feel that they were being asked to do extra work.

Figure 1

MIC menu for measurement unit

Integrating Mathematics and Science Measuring What You Observe Science is all about being able to communicate what you observe. Measure water using different measuring tools and think about what is the best tool for the job!	Problem Solving A Horse Has Got to Eat Build a fence for your hungry horse that lets him eat the most grass.	Writing about Mathematics A Citizen Writes to the President Write a letter to the president telling him which system of measurement (metric or U.S. customary) Americans should use and why!
Mathematics Game Poison Figure out how *not* to be the person who takes the last penny!	Data Project Walking a Million Steps Find out how far you would have walked if you walked a million steps!	Logic Problem Treachery Help Christopher Columbus solve the mystery of how fast he should go in order to prevent Captain Devious from becoming the first one to sail to San Salvador Island!
Mathematics and Social Studies Accuracy on the Appalachian Trail Working with an enlarged map, use Cuisenaire rods to measure the part of the Appalachian Trail that runs through Virginia.	Building Project The Dunking Booth Challenge Can you measure out exactly 4 cups of water using only a 3-cup jar and a 5-cup jar? Solve this challenge so that you can dunk Ms. Lester!	Literature and Mathematics Crazy Measuring Read *Counting on Frank* (Clement 1994), then make up your own crazy measuring problem!

The MIC was available to any students whom the teacher felt would benefit from more challenging mathematics activities, not just students identified as gifted. One or two of the activities in each unit were appropriate for the whole class and provided enrichment for everyone at some level. However, the intent of the MIC was to provide activities for the gifted students, and we soon discovered that most of the activities we designed were too difficult for the majority of the students without intensive support and assistance.

Student menu guide

For each unit, a menu of center choices was created and laid out in a 3×3 grid using Winebrenner's format (2001). **Figure 1** presents a sample menu for a third-grade measurement unit. The students were usually given the freedom to select any activity in the center that they wished. At other times, the teacher asked this mathematics group to work on a certain activity. If a teacher wanted to guide the students' choices, he or she could set up the menu strategically and ask students to select activities according to a particular rule (Winebrenner 2001). For example, students could be asked to do "three in a row," "four corners," or "the middle, plus any four outer activities." Occasionally, activities built on one another in such a way that one needed to be completed before another.

Types of activities

All nine types of activities in the MIC were related to a theme that an elementary teacher might be covering in a given unit in her class (such as multiplication, measurement, geometry, and so on). Although the activities themselves changed from unit to unit, the nine types remained the same across all units. Four were intended to stretch mathematics across the curriculum (Integrating Mathematics and Science, Writing about Mathematics, Literature and Mathematics, and Mathematics and Social Studies); these provided enrichment by making connections to other subjects. Four other types of activities were intended to develop logical thinking and problem-solving skills (Mathematics Game, Building Project, Logic Problem, and Problem Solving); these activities provided enrichment by creating opportunities to focus on processes of mathematics. The ninth type of activity was based on the use of data, in which the students were asked to collect, tally, and represent findings from data they collected themselves (Data Project); this

Figure 2

Nine activity types

Mathematics across the Curriculum

1. Integrating Mathematics and Science: This activity is a short science experiment or research project within which a mathematics problem is embedded. It is not just a mathematics problem with science words. The closer it is related to what the class is currently studying, the better.
2. Writing about Mathematics: The idea is to devise an activity in which children have to write about their mathematical thinking processes so their thinking becomes more deliberate. This activity is a writing activity, not just documentation of what happened in a problem.
3. Mathematics and Social Studies: This activity is a mathematics problem or activity that is contextually based in everyday family, cultural, or citizenship activities. The more it is truly an activity a teacher would do in social studies and the more related it is to what the class is currently studying, the better.
4. Literature and Mathematics Connection: This activity consists of having the children read literature and then asking a question about the mathematics content. Children's literature with mathematics content can also be used to elaborate on a mathematics concept or illuminate a problem situation. Many books are available to guide you in choosing literature for this activity. Depending on the difficulty of the book or its mathematics content, this activity can be good for sharing with the whole class, not just gifted students.

Logical Thinking and Problem Solving

5. Mathematics Game: Children have fun playing games with their friends. Lots of good practice in basic skills can be embedded in mathematics games, but problem-solving skills can also be developed while strategizing. This activity often turned out to be one in which the whole class could participate in playing but which the gifted students could be encouraged to analyze and strategize.
6. Logic Problem: This can be any activity that requires logical thinking. Fair-share games, matrix logic puzzles, table logic puzzles, or Venn diagram logic puzzles are all excellent choices for which reproducible activities are easily available.
7. Building Project: This activity is intended to appeal to the kinesthetic learner. The kind of activity could range from paper folding to block building to collage making. This is an excellent opportunity to design an activity that uses manipulatives.
8. Problem Solving: Although almost every activity in this framework is a problem to solve, teachers can use this square to specifically target different problem-solving strategies over the course of the school year by creating problems that are most efficiently solved using a particular strategy.

Data Project

9. Data Project: Collecting and tabulating data provide rich opportunities for children to use their mathematical skills and learn something interesting. Organizing data to make it easier to see patterns and trends is an important skill in problem solving. One potential for this kind of problem is long-term project work in which children might collect data over a period of time.

provided enrichment by encouraging students to organize mathematical information and look for patterns. **Figure 2** describes each of these types of activities in more detail.

Characteristics of individual activities

The activities were the cornerstone of differentiating the curriculum for gifted mathematics students and focused on problem solving, cross-curricular connections, and processes of mathematics. Using the same mathematical theme that the rest of the class was studying, the activities provided depth for the gifted students by shifting from a computation level to a problem-solving level. Forging connections to other content areas of the curriculum also provided depth to the students' mathematics experiences. Processes of mathematics became the focus of many of the activities, so we chose to ground our activities in an investigative approach (Baroody 1998) in which the process was the critical element.

Principles and Standards (NCTM 2000) outlines five broad types of processes: problem solving, reasoning and proof, communication, connections, and representation. Individually, each activity met one or more of the Process Standards. Taken as a whole unit, the nine activities gave students the opportunity to develop all these processes.

Selecting, modifying, and creating activities

Everyday Mathematics (by the University of Chicago School Mathematics Project) is the adopted curriculum for our county; however, not enough challenging activities were available in this curriculum to meet the gifted students' needs. Therefore, we selected individual activities from a variety of sources, including Everyday Mathematics curriculum suggestions, puzzle books, gifted education resources, literature and mathematics lists, Web sites, and mathematics education texts. We modified some activities because we wanted to make

Figure 3

Ten criteria for selecting, modifying, and creating activities

1. The activity is investigative and will require some initiative and discovery on the student's part. Recipe-like activities in which the students are walked through the steps of one solution are to be specifically avoided.
2. The activity can be approached in different ways. There may be more than one pathway to the solution or more than one solution.
3. The activity is complex and will require a variety of mathematical skills to solve. The activity may or may not be solvable in one mathematics period, allowing students to learn to cope with frustration and develop perseverance.
4. The activity is structured so that gifted students of a variety of abilities can begin the problem at their own level. However, the activity is *not* designed to be simple enough that the lowest-achieving student can succeed without assistance.
5. The activity will provide practice or fresh insight into the skills being presented in the regular mathematics unit.
6. The activity is engaging for elementary school students.
7. The activity is structured so that it can be worked on individually or in small groups, thus providing opportunities to discuss mathematical ideas.
8. The activity is structured to encourage reflection and communication about mathematical ideas.
9. For each unit, attention will be given to different learning styles. For example, some of the nine activities will be geared toward kinesthetic learners, others to visual learners, and so on.
10. For each unit, attention will be given to Bloom's taxonomy. For example, some of the nine activities will be designed to promote analysis, synthesis, or evaluation. Comprehension is a prerequisite, not a goal, of all activities.

them more discovery-oriented. We created others from scratch because we wanted them to coordinate closely with another lesson (such as social studies or science).

Our criteria for selecting, modifying, and creating activities were based on suggestions made by Sheffield (2003) and Baroody (1998). Tomlinson (1999) also spells out guidelines for differentiation that match the goals we had when creating activities. See **figure 3** for details.

The specific activities for each unit were selected, modified, or created to be "worthwhile mathematical tasks" (NCTM 2000, pp. 18–19) and were intended to be seeds for a good mathematical investigation experience. According to *Principles and Standards*, this type of task bundles mathematical thinking, concepts, and skills; sparks students' curiosity; and encourages speculation and investigation. Baroody (1998) notes that a worthwhile task can be approached in different ways and that it leads naturally to mathematical discussion. We deliberately tried to avoid activities that were structured like recipes or that walked the students through unfamiliar territory step by step. Instead, we chose inquiry-based activities.

By design, most activities had more than one solution path, so there was no answer key. We required that solutions to the activities be written so that the teacher could understand what the solution was and how the student figured it out. Having the students answer questions along the way demonstrated their thought processes as they worked toward a solution. Sheffield (2003) states that students explore problems in depth when they go beyond a solution to consider generalizations, comparisons, and relationships to other mathematical situations. Questions embedded in the activity push students in this direction. **Figure 4**, A Horse Has Got to Eat, lists important components of a problem intended for third-grade gifted mathematics students.

Assessment

Students' work on the activities at the Mathematics Investigation Center could be assessed by using multiple criteria including time on task, finding a workable solution to the problem posed in an activity, and communicating thought processes orally or in writing to the teacher. Tomlinson (1999) points out that teachers use assessment procedures to assign grades but also to determine whether the work is appropriately challenging for the student. Depending on the assessment goal, a teacher could choose one or more criteria for evaluating students' work.

Evaluative criteria may also depend on how the teacher presents the MIC work to the students. If students expect to work at the MIC because that is what their mathematics group does, then more specific feedback to the students about how well they are accomplishing the tasks may be warranted. However, if the students perceive work at the MIC as a privilege or as extra work, then they may find ways to sabotage themselves at the center in order to return to work where they get the easy A. In this case, Winebrenner (2001) suggests that students be given an A for all work they have tested out of and be given a daily effort grade reflecting their diligence

Figure 4

Problem solving: A Horse Has Got to Eat

Unit: Measurement[a]
Materials: Graph paper, string[b]
Introduction: You love horses,[c] and your parents have just bought you one! Your neighbor has agreed to let you build a fence in her field to let your horse graze. You have only one piece of electric fence wire 100 feet long, and you have as many stakes as you need to hold the fence wire. Because the fence is electric, assume that you need just one strand of fence all the way around your horse. Build a fence around your horse with this wire.[d]

1. Draw a picture of your fence on a piece of graph paper.[e] Make sure your picture shows that your wire is 100 feet long.
2. What shape is the fence you made?[f]
3. How can you use the string to represent the 100 feet of wire?
4. Your horse is very hungry! Experiment with different shapes to solve this problem[g] so that when you build the fence around your horse, the horse is able to eat the most grass.[h]
5. What shapes are your fences?[i]
6. Draw pictures of your fences that show how you know that your horse will eat the most grass in the fenced area you drew.[j]
7. Is this problem similar to another kind of problem you have worked on before? Explain the similarity.[k]

a. This is the theme of the unit the teacher was teaching in mathematics.
b. This activity will appeal to a kinesthetic learner and can be done alone or in pairs.
c. The fantasy of owning a horse may motivate some elementary school students to dive into the problem.
d. There will be more than one solution to this problem, allowing students to approach it in their own way.
e. The use of graph paper (and later, string) addresses NCTM's Standards of Representation and Communication and helps students think about scale.
f. A direct question helps the students write down their thoughts for the teacher. This particular question about shapes highlights the way in which a complex problem touches on several areas of mathematics at once (in this case, geometry, measurement).
g. A direct suggestion for how to approach the problem may be necessary if you want to see evidence of trial-and-error problem solving in the write-up. If you do not, this language can be omitted, making the problem more open.
h. This is a more difficult problem to solve. It will offer opportunities for abstract thinking about the properties of shape and area and give the students practice in representing measurements if they try out several solutions.
i. You may choose to accept only solutions that are circles, or you may choose to accept the "best" shape of the ones the student tried. Some students may even begin to articulate a conceptualization of limits and infinity.
j. These are more opportunities to communicate and represent mathematical ideas.
k. This probe encourages the students to connect their solution to other things they have done in mathematics or other domains. By comparing and contrasting, the students move beyond comprehension to an analysis level of Bloom's taxonomy. Implicitly, the question says that the learning is not over when an answer to the problem is found.

at the MIC, instead of having the more challenging work influence their grade-point average.

Responding to the Needs of Gifted Children

Enrichment

Gifted students can benefit from opportunities to grapple with worthwhile tasks because gifted students think differently from other students (Strip 2000). On the bell curve of IQ scores, gifted students are as different from their age-mates as they are from students whose IQs fall below 85 (Winebrenner 2001). Unlike many of their age-mates, for example, gifted students have an ability to work with abstract or complex concepts; to be enriched, they need activities that move beyond a comprehension level. Gifted students tend to make more rapid progress through new material and have an outstanding memory for information; to be enriched, they need to be able to pace themselves, and they do not need much review or practice. Gifted students may approach tasks in unique ways, sometimes because they see a new way to accomplish the task or a connection to another process that gives them insight; to be enriched, they need opportunities for creativity and independence. In short, gifted students do not need more work of the same type that is offered in many traditional curricula; instead, they need different types of activities (Galbraith 2001).

Semi-independence

Students who are gifted in mathematics need both teacher direction and opportunities for independence. These students are sometimes perceived as not needing help with mathematics because they can accomplish the mainstream curriculum without much assistance. When we challenge them to apply concepts in new ways or grapple with problems that take them in new directions, however, we cannot expect them to learn entirely on their own. Gifted students also need opportunities to develop personality attributes such as creativity, curiosity, insight, perseverance, and imagination (Piirto 1998). These attributes may be best developed when students are given opportunities to struggle with complex problems on their own. A teacher in a facilitator role leaves students alone when appropriate and offers instruction or assistance when appropriate.

An important area in which gifted students may need help is motivation. Although many gifted students are capable of sustained attention and focus on a task when it is of interest to them, the intrinsic motivation may not exist when we challenge gifted students to do more difficult work than they are accustomed to doing. Teacher-facilitators can assist students with staying on task and applying themselves fully to the problems. Also, many gifted students struggle with perfectionism when they are asked to work on problems for which they cannot see an immediate, clear-cut solution (Smutny 2001; Strip 2000; Winebrenner 2001). A teacher-facilitator can be supportive to gifted students by helping them learn self-discipline, take risks, and develop tolerance for ambiguity.

Gifted students need some instruction and guidance. Yet teachers are asked to work with a range of ability levels in their classrooms and cannot always provide instruction to all groups at once. Using a center approach, in which the teacher acts as a facilitator rather than a direct instructor and in which students work in pairs or small groups at times, allows gifted mathematics students to work independently with support. We call this semi-independence.

In the field

The Mathematics Investigation Center started small but is growing. This framework was conceived by a parent looking for ways to help her child, organized by a gifted education specialist committed to facilitating any efforts to help gifted children in her school district, implemented by a pioneering third-grade teacher looking for ways to serve higher-level mathematics students in her classroom, and advised by a university mathematics educator. Last year this small collaboration was picked up by third-grade teachers across the school district who refined and extended the Mathematics Investigation Center to cover the whole year's worth of curriculum. They were joined by fifth-grade teachers across the district trying to accomplish the same thing.

The MIC framework is generic enough that each teacher can implement the center in accordance with his or her classroom management style. The menu and activity descriptions seem to help start the process of planning for differentiation in mathematics. For example, this year the first- and fourth-grade teachers in the district have picked up the framework and are planning to use a version of it in their classrooms. Initially, there were nine activities; however, the fourth-grade teachers in our county have chosen to use a seven-activity "pie chart" menu with six activities around a circle and one activity in the center. The nine types of activities have been reserved to rotate into the seven slots, and a tenth type of activity, Math in the Media, has been proposed.

Conclusion

"I don't want things to be simple!" states third-grader Kiana after the adult volunteer suggests she try a less complicated shape. She is creating a highly irregular polygon "fence" with beautiful symmetry for the problem A Horse Has Got to Eat. Figuring out the area and perimeter of the shape she has drawn will be tough, but she is enjoying her own creativity and would rather struggle a bit before looking for an easier path to the solution. She has already come up with several shortcuts to replace counting all the squares to determine the area of her polygon. The adult volunteer looking at her drawing asks how long the sides of the shape are where they cut diagonally across a graph-paper square and suggests that Kiana test her assumption that the diagonals are the same length as the sides. Using a measuring tape, Kiana discovers that the diagonals of the graph-paper squares are longer than the sides of the squares, making the perimeter less straightforward to determine than a rectangle. At times, she is frustrated. In fact, she eventually decides to start over with a simpler polygon but still refuses to use a rectangle. Working through this problem and the snags she encounters, Kiana deepens her understanding of area and perimeter and also strengthens her problem-solving skills and her perseverance. Because she already comprehends how area and perimeter are defined and because she has an ease with the arithmetic skills she needs to compute the solutions, this is a challenging problem for her (and other gifted mathematics students) that is neither overwhelming nor too simple.

Differentiating the mathematics curriculum for gifted students is an important task worth doing. Kiana's real-life experience with the A Horse Has Got to Eat problem demonstrates how a complex problem can provide opportunities for gifted children to deepen their mathematical understandings of concepts they already comprehend well enough to pass a test. Students who are gifted in mathematics are short-changed when they are not given appropriate levels of mathematics work; they may misbehave in class or begin to lose interest in a topic that they used to enjoy. Worst of all, they may begin to perceive mathematics as a subject in which they do not have to struggle to succeed, a subject that they finish as quickly as possible. Without appropriate opportunities, students with great potential in mathematics, such as Carlos and Kiana, may never develop the skills, motivation, and perseverance they need to reach their potential.

Differentiating the mathematics curriculum may be a challenge to teachers, but it is possible. The pressure on elementary teachers to bring as many students up to grade level as needed to pass nationally or state-mandated standardized tests is strong. The No Child Left Behind act may have weakened the pressure to differentiate the curriculum for gifted students. A teacher can use a Mathematics Investigation Center with a few well-chosen mathematical investigation activities to enrich the theme of the regular curriculum for weeks at a time instead of having to plan a second or third set of lessons every day. Our hope is that the framework we have outlined simplifies the task of differentiation, making it more manageable for elementary school teachers.

References

Baroody, Arthur J., with Ron T. Coslick. *Fostering Children's Mathematical Power: An Investigative Approach to K–8 Mathematics Instruction*. Mahwah, NJ: Lawrence Erlbaum Associates, 1998.

Clement, Rod. *Counting on Frank.* Boston: Houghton Mifflin School Division, 1994.

Cohen, Peter A. "Educational Outcomes of Tutoring: A Meta-analysis of Findings." *American Educational Research Journal* 19, no. 2 (1982): 237–48.

DeLacy, Margaret. "The 'No Child' Law's Biggest Victim? An Answer That May Surprise." *Education Week* 23, no. 41 (2004): 40.

Galbraith, Judy. *The Gifted Kids' Survival Guide for Ages 10 and Under.* Minneapolis, MN: Free Spirit Publishing, 1998.

National Council for Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics*. Reston, VA: NCTM, 2000.

Piirto, Jane. *Understanding Those Who Create.* 2nd ed. Scottsdale, AZ: Great Potential Press, 1998.

Sheffield, Linda J. *Extending the Challenge in Mathematics: Developing Mathematical Promise in K–8 Students*. Thousand Oaks, CA: Corwin Press, 2003.

Smutny, Joan F. *Stand Up for Your Gifted Child: How to Make the Most of Your Kids' Strengths at School and at Home*. Minneapolis, MN: Free Spirit Publishing, 2001.

Strip, Carol A., with Gretchen Hirsch. *Helping Gifted Children Soar: A Practical Guide for Parents and Teachers*. Scottsdale, AZ: Gifted Psychology Press, 2000.

Tomlinson, Carol A. *The Differentiated Classroom: Responding to the Needs of All Learners.* Alexandria, VA: Association for Supervision and Curriculum Development, 1999.

Winebrenner, Susan. *Teaching Gifted Kids in the Regular Classroom: Strategies and Techniques Every Teacher Can Use to Meet the Academic Needs of the Gifted and Talented*. Minneapolis, MN: Free Spirit Publishing, 2001.

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